al-Farabi Kazakh National University

Faculty of Mechanics and Mathematics

Mathematics Department

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|  | APPROVED  Dean of the Faculty of Mechanics and Mathematics \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ D. Zhakebaev  "\_\_\_"\_\_\_\_\_\_\_\_\_2020 |

EDUCATIONAL COMPLEX FOR DISCIPLINE

"Mathematical modelling"

Specialty - Mathematics (5B060100)

Course - 4

Semester - 7

Number of credits - 3

Almaty 2020

The educational-methodical complex was developed by Doctor of Physical and Mathematical Sciences, Professor S.Ya. Serovajsky.

Developed on the basis of the curriculum for the specialty 5B060100 – Mathematics

Reviewed and recommended at a meeting of the Department of Mathematics

"\_\_\_" \_\_\_\_\_\_\_\_\_\_\_\_\_\_ 2020, protocol No. \_\_\_.

Head of the Department \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Kh. Khompysh

Recommended by the methodical bureau of the faculty

"\_\_\_\_" \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, 2020, protocol No. \_\_\_\_\_.

Head of the Methodology Bureau of the Faculty \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Syllabus**

**Course Information**

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| Discipline code | Discipline name | | Class | Number of hours per week | | | | credit numbers | | IWST |
| Lectures | Seminars | | Lab. |
|  | Mathematical modelling | |  | 2 | 1 | | - | 3 | |  |
| Lecturer | | S. Ya. Serovajsky, doctor of science, professor | | | | Class time | | | Scheduled | |
| e-mail | | [serovajskys@mail.ru](mailto:serovajskys@mail.ru) | | | |
| phone | | +7 701 8315197 | | | | lecture hall | | |  | |
| assistant | |  | | | | Working hours | | | Scheduled | |
| e-mail | | E-mail: | | | |
| phone | |  | | | | lecture hall | | |  | |

**Academic policy**

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| **Purpose of the discipline** | **Expected learning outcomes (LO)**  As a result of studying the discipline, the student will be able to: | **LO Achievement Indicators (AI)**  (for each LO at least 2 indicators) |
| Studying the general principles of constructing mathematical descriptions of various processes and methods of their analysis | LO1. Have a basic understanding of mathematical modeling | AI1.1. Know the general principles of building mathematical models;  AI1.2. Know the characteristics of mathematical models |
| LO2. Know the basic applications of lumped systems. | AI2.1. Be able to describe physical processes.  AI2.2. Be able to describe chemical processes.  AI2.3. Be able to describe the processes of biology and medicine.  AI2.4. Be able to describe the processes of economics and social sciences |
| LO3. Know the basic applications of distributed parameter systems. | AI3.1. Know basic mathematical descriptions of transfer processes.  AI3.2. Know basic mathematical descriptions of transfer processes.  AI3.3. Know basic mathematical descriptions of stationary systems |
| LO4. Be able to understand the variational principles and problems of correctness, control and identification. | AI4.1. Have an understanding of the principles of variation.  AI4.2. Have an understanding of the system correctness problem  AI4.3. Be able to solve systems management problems  AI4.4. Be able to solve problems of Identification of systems |

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| Prerequisites | Differential equations, equations of mathematical physics, numerical methods, physics, calculus of variations |
| Post-requisites | Special courses |
| Informational resources | 1. S.Serovajsky. Mathematical modeling. - Almaty: Kazakh University, 2000. 2. S. Serovajsky. Mathematical modeling. - London, CRC Press, 2021 (to appear). 3. V.S. Zarubin. Mathematical modeling in technology. - M., 2003. 4. L.A. Anchordoqui, T.C. Paul. Mathematical Models of Physics Problems. – Nova Science Publishers, 2013 5. H. Gould, J. Tobochnik, W. Christian. An introduction to computer simulation methods: Applications to Physical Systems. – Pearson/Addison Wesley, 2006. 6. M. Meerschaert. Mathematical Modeling. – Academic Press, 2007. 7. S. Moghadas, M. Jaberi-Douraki. Mathematical Modelling: A Graduate Textbook. – Wiley, 2018. 8. <https://people.maths.bris.ac.uk/~madjl/course_text.pdf> 9. <http://www.sfu.ca/~vdabbagh/Chap1-modeling.pdf>   <https://en.wikipedia.org/wiki/Mathematical_model> |
| Academic policy of the course in the context of university values | Rules of academic conduct: Compulsory attendance at classes, inadmissibility of delays, adherence to Deadline s for completing and delivering assignments (IWS, seminars, midterm exam, individual projects).  Academic values: According to article 5 of the Code of honor of a student of the al-Farabi Kazakh National University, the student must strictly fulfill his academic duties and not allow academic and legal violations (plagiarism, forgery, the use of cheat sheets, deceiving the teacher and disrespectful attitude towards him, truancy and being late without good reason).  All students can receive advice in person, at the indicated phone numbers or via e-mail. |
| Assessment and attestation policy | Criteria assessment: evaluating the results in accordance with the descriptors (checking the formulation of competencies during the weeks of midterm control, intermediate and final examinations)  Summative assessment:  Final assessment of the discipline =  MC1, MC2 – midterm control, МТ – midterm, ИК – final control.  Percentage-rating letter system for assessing educational achievements of students:  95% - 100%: А 90% - 94%: А-  85% - 89%: В+ 80% - 84%: В 75% - 79%: В-  70% - 74%: С+ 65% - 69%: С 60% - 64%: С-  55% - 59%: D+ 50% - 54%: D- 0% -49%: F |

**CALENDAR OF IMPLEMENTATION OF THE CONTENT OF THE TRAINING COURSE:**

Contents

L - lecture; S - seminar; SE - questions for self-examination; IT - individual tasks, MC - midterm control.

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| week / module | Topic name | LO | AI | Number of hours | Maximum score | | Knowledge assessment form |
| **Module I. Introduction** | | | | | | | |
| 1 | L1. Cognition and modeling. Natural science and mathematics. Body fall equation. Principles of building a mathematical model. Classification of mathematical models. | LO1 | AI1.1 | 2 | 3 | | SE1 |
| S1. Characteristics of mathematical models of flight dynamics. | LO1 | AI1.2 | 1 | 5 | | IT1 |
| IT1. Characteristics of mathematical models of flight dynamics. | LO1 | AI1.2 |  | 12 | |  |
| **Saturday 18.00 - Deadline SE1, IT1, IT1** | | | | | | |
| **Module II. Lumped-parameter systems** | | | | | | | |
| 2 | L2. Mathematical models of mechanical oscillations (Derivation of the equation of the pendulum oscillation. Solution of the pendulum oscillation equation. Pendulum oscillation energy. Pendulum oscillation in the presence of friction. Forced pendulum oscillations). | LO2 | AI2.1 | 2 | 3 | | SE2 |
| S2. Spring oscillation modelling. | LO2 | AI2.1 | 1 | 5 | | IT2 |
| IT2. Spring oscillation modelling. | LO2 | AI2.1 |  | 12 | |  |
| **Saturday 18.00 - Deadline SE2, IT2, IT2** | | | | | | |
| 3 | L3. Mathematical models of electrical oscillations (Electric circuit. Energy of the circuit. Circuit with resistance. Forced oscillations of the circuit). | LO2 | AI2.1 | 2 | 3 | | SE3 |
| S3. Analogy between electrical and mechanical vibrations. | LO2 | AI2.1 | 1 | 5 | | IT3 |
| IT3. Analogy between electrical and mechanical vibrations. | LO2 | AI2.1 |  | 12 | |  |
| **Saturday 18.00 - Deadline SE3, IT3, IT3** | | | | | | |
| 4 | L4. Mathematical models of chemistry (Equations of chemical kinetics. Monomolecular reaction. Bimolecular reaction. Volterra-Lotka system). | LO2 | AI2.2 | 2 | 3 | | SE4 |
| S4. Mathematical modeling of chemical reactions. | LO2 | AI2.2 | 1 | 5 | | IT4 |
| IT4. Mathematical modeling of chemical reactions. | LO2 | AI2.2 |  | 12 | |  |
| **Saturday 18.00 - Deadline SE4, IT4, IT4** | | | | | | |
| 5 | L5. Mathematical models in biology. (Models of Malthus and Verhulst. Model "biological competition". Model "predator-prey". Model "symbiosis"). | LO2 | AI2.3 | 2 | 3 | | SE5 |
| S5. Mathematical models of the coexistence of several biological species. | LO2 | AI2.3 | 1 | 5 | | IT5 |
| IT5. Mathematical models of the coexistence of several biological species. | LO2 | AI2.3 |  | 12 | |  |
| **Saturday 18.00 - Deadline SE5, IT5, IT5** | | | | | | |
| **MC1** | | | | **100** | | |
|  | | | | | | | |
| 6 | **Л6.** Математические модели в экономике (Развитие одной фирмы. Модель "экономическая конкуренция". Модель "экономическая ниша". Модель Солоу). | LO2 | AI2.4 | 2 | 3 | | SE6 |
| **S6.** Аналогия между биологическими и экономическими моделями. | LO2 | AI2.3 AI2.4 | 1 | 5 | | IT6 |
| **IT6.** Аналогия между биологическими и экономическими моделями. | LO2 | AI2.3 AI2.4 |  | 12 | |  |
| **Saturday 18.00 - Deadline SE6, IT6, IT6** | | | | | | |
| 7 | L7. Mathematical models in political science, sociology and psychology (Model of political competition. Model "political niche". Model of a two-party system. Model of trade union activity. Model of family relations.) | LO2 | AI2.4 | 2 | 3 | | SE7 |
| S7. Analogy between lumped dynamical systems. | LO2 | AI2.3 AI2.4 | 1 | 5 | | IT7 |
| IT7. Analogy between lumped dynamical systems. | LO2 | AI2.3 AI2.4 |  | 12 | |  |
| **Saturday 18.00 - Deadline SE7, IT7, IT7** | | | | | | |
| 8 | L8. Mathematical models of epidemiology (Models SIR, SEIR, SEIRD, SIS). | LO2 | AI2.3 | 2 | 3 | | SE8 |
| S8. Compilation of mathematical models of epidemiology | LO2 | AI2.2 | 1 | 5 | | IT8 |
| IT8. Compilation of mathematical models of epidemiology | LO2 | AI2.3 |  | 12 | | IT3 |
| **Saturday 18.00 - Deadline SE8, IT8, CРC8** | | | | | | |
| **Module III. Distributed parameter systems** | | | | | | | |
| 9 | L9. Mathematical modeling of transport processes (Equation of heat conduction. Statement of boundary value problems. The first boundary value problem for the homogeneous equation of heat conduction. Inhomogeneous equation of heat conduction). | LO3 | AI3.1 | 2 | 3 | | SE9 |
| S9. Analogy between models of transfer processes. | LO3 | AI3.1 | 1 | 5 | | IT9 |
| IT9. Analogy between models of transfer processes. | LO3 | AI3.1 |  | 12 | | IT4 |
| **Saturday 18.00 - Deadline SE9, IT9, IT9** | | | | | | |
| 10 | L10. Mathematical modeling of wave processes (Derivation of the string vibration equation. Boundary value problem statement. String vibration energy. Traveling waves. String vibration with fixed ends). | LO3 | AI3.2 | 2 | 3 | | SE10 |
| S10. Analogy between electrical and mechanical wave processes. | LO3 | AI3.2 | 1 | 5 | | IT10 |
| IT10. Analogy between electrical and mechanical wave processes. | LO3 | AI3.2 |  | 12 | | Письменная КР |
| **Saturday 18.00 - Deadline SE10, IT10** | | | | | | |
| **Midterm** | | | | **100** | | |
| 11 | L11. Mathematical models of stationary systems (Stationary heat transfer. Stationary fluid flow. Gravitational field. Electrostatic field. Statement of boundary value problems). | LO3 | AI3.3 | 2 | 3 | | SE11 |
| S11. Analogy between gravitational and electrostatic fields. | LO3 | AI3.3 | 1 | 5 | | IT11 |
| IT11. Analogy between gravitational and electrostatic fields. | LO3 | AI3.3 |  | 12 | |  |
| **Saturday 18.00 - Deadline SE11, IT11, IT11** | | | | | | |
| **Module IV. Addition** | | | | | | | |
| 12 | L12. Variational principles (The principle of least action and the motion of a material point. Fermat's principle and the laws of refraction of light. Variational problems with constraints and the equation of oscillation of a pendulum. Multidimensional variational problems and the equation of oscillation of a string). | LO4 | AI4.1 | 2 | 3 | | SE12 |
| S12. Variational principles in problems of mechanics. | LO4 | AI4.1 | 1 | 5 | | IT12 |
| IT12. Variational principles in problems of mechanics. | LO4 | AI4.1 |  | 12 | |  |
| **Saturday 18.00 - Deadline SE12, IT12** | | | | | | |
| 13 | L13. Mathematical models and the problem of correctness (Cauchy problem for differential equations. Boundary value problems for differential equations. Hadamard's example. Correct and ill-posed problems for the heat equation. Euler's problem on an elastic rod). | LO4 | AI4.1 | 2 | 3 | | SE13 |
| S13. Examples of correct and incorrect problems. | LO4 | AI4.2 | 1 | 5 | | IT13 |
| IT13. Examples of correct and incorrect problems. | LO4 | AI4.2 |  | 12 | | IT5 |
| **Saturday 18.00 - Deadline SE13, IT13, IT13** | | | | | | |
| 14 | L14. Mathematical modeling and optimal control problems (Control and optimization. Maximizing the flight range of the body. Formulation of the optimal control problem. Maximizing the flight range of a guided missile). | LO1 | AI4.3 | 2 | 3 | | SE14 |
| S14. Examples of optimal control problems. | LO4 | AI4.3 | 1 | 5 | | IT14 |
| IT14. Examples of optimal control problems. | LO4 | AI4.3 |  | 12 | | IT6 |
| **Saturday 18.00 - Deadline SE14, IT14, IT14** | | | | | | |
| 15 | L15. Identification of mathematical models (the problem of identifying mathematical models in examples. Direct and inverse problems of heat conduction. Ill-posedness of inverse problems. Solving inverse problems using optimization methods). | LO4 | AI4.4 | 2 | 3 | | SE15 |
| S15. Relationships between inverse and optimization problems. | LO4 | AI4.4 | 1 | 5 | | IT15 |
| IT15. Relationships between inverse and optimization problems. | LO4 | AI4.4 |  | 12 | |  |
| **Saturday 18.00 - Deadline SE15, IT15, IT15** | | | | | | |
| **MC2** | | | | | **100** | |

Dean D. Zhakebaev

Methodical bureau chairman

Department head Kh. Khompysh

Lecturer S. Serovajsky

**DIVISION OF DISCIPLINE INTO KNOWLEDGE BLOCKS**

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| --- | --- | --- | --- |
|  | 1. BASIC KNOWLEDGE | 2. FUNCTIONAL KNOWLEDGE | 3. SYSTEM KNOWLEDGE |
| 1. Introduction | The concept of a mathematical model. Building models. Model characteristics. Classification of models | Characterization of specific mathematical models. | Modeling as a form of cognition. The connection of mathematics with the outside world. |
| **2. Lumped-parameter systems** | Mathematical models of processes in physics, chemistry, biology, economics, sociology. | Construction of mathematical models of dynamic processes of various sciences and their study | Mathematical description of various processes |
| **3. Distributed parameter systems** | Transfer processes, wave processes and stationary systems | Construction of mathematical models of distributed systems and their study | Mathematical description of distributed systems |
| **4.** **Supplements** | Variational principles, correctness, control and identification of mathematical models | Description of processes using variational principles, assessment of the correctness of systems, optimization based on mathematical modeling, inverse problems. | Connection of mathematical modeling with various areas of mathematics. Adaptation of models to specific situations. |